Sheet 2

Problem 1: *Recursive Factorial*

The factorial of a nonnegative integer n is written as $n!$ (pronounced "n factorial") and is defined as follows:

$$n! = \begin{cases} n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1 & , n \geqslant 1 \\ 1 & , n = 0 \end{cases}$$

For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, which is 120.
Write a recursive method `fact` that takes a nonnegative integer and returns its factorial.

Problem 2: *GCD*

The greatest common divisor (GCD) of two integers is the largest integer that evenly divides each of the numbers.

For example, $\gcd(42, 12) = 6$.

Write a method `gcd` that takes two nonnegative integers and returns their greatest common divisor.

a) Using Brute Force

b) Using division-based version of *Euclid* Algorithm:

$$gcd(m, n) = \begin{cases} gcd(n, m \mod n) & , n \geqslant 1 \\ m & , n = 0 \end{cases}$$

i. with recursion

ii. with iteration

c) Using subtraction-based version of *Euclid* Algorithm (with recursion):

$$gcd(m, n) = \begin{cases} gcd(m-n, n) & , m > n \\ gcd(m, n-m) & , m < n \\ m & , m = n \end{cases}$$

Problem 3: *Standard Deviation*

Write a method `sigma` that takes a one dimensional array of real numbers and returns the standard deviation σ.

a) Using

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}, \mu = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

b) Using

$$\sigma = \sqrt{\mathrm{E}[X^2] - (\mathrm{E}[X])^2}, E[X] = \frac{1}{N}\sum_{i=1}^{N} x_i, E[X^2] = \frac{1}{N}\sum_{i=1}^{N} x_i^2.$$